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$$1-a^2=[1-2(y/r)^{\frac{2}{3}}]/[1-(y/r)^{\frac{2}{3}}].$$

$$a/(1-a^2)=(y/r)^{\frac{1}{3}}\sqrt{[1-(y/r)^{\frac{2}{3}}]}/[1-2(y/r)^{\frac{2}{3}}] \dots\dots\dots(3).$$

Substitute (2) and (3) in (1),

$$y=\left(x-\frac{r}{2\sqrt{[1-(y/r)^{\frac{2}{3}}]}}\right)\times\frac{2(y/r)^{\frac{1}{3}}\sqrt{[1-(y/r)^{\frac{2}{3}}]}}{1-2(y/r)^{\frac{2}{3}}};$$

$$x=\frac{y[1-2(y/r)^{\frac{2}{3}}]}{2(y/r)^{\frac{1}{3}}\sqrt{[1-(y/r)^{\frac{2}{3}}]}}+\frac{r}{2\sqrt{[1-(y/r)^{\frac{2}{3}}]}};$$

or $x=\frac{1}{2}(r^{\frac{2}{3}}+2y^{\frac{2}{3}})\sqrt{(r^{\frac{2}{3}}-y^{\frac{2}{3}})}$, the equation sought.

60. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

When the sun's declination is $23^{\circ} 27' 15''$ North= δ , in what latitude will it shine on the *north* side of buildings during the first half of the forenoon, and on the *south* side during the other half, and what will be the length of the day?

I. Solution by the PROPOSER.

Let P be the north pole, PA a meridian, Z the zenith, HS the horizon, the sun rising at S , and describing the small circle arc SBA , during the forenoon, reaching A at apparent noon, ZH a prime vertical, bisecting the semi-diurnal arc SBA at B , and right-angled at Z , the declination= δ , and latitude= λ , $\tan\delta=\alpha$, and $\tan\lambda=\chi$.

Then $PB=90^{\circ}-\delta$, $PZ=90^{\circ}-\lambda$, and $\angle ZPB=h$ =the hour-angle at the pole, and measures the time of describing the arc BA on the south side of ZH .

$$\cosh=\tan\delta\cot\lambda=\alpha\chi.$$

The hour angle measuring $SBA=2h$, and $\cos 2h=-\tan\delta\tan\lambda=-\alpha\chi$. But $\cos 2h=(2\alpha^2/\chi^2)-1=-\alpha\chi$. $\therefore \chi^3-(\chi^2/\alpha)+2\alpha=0$. Solving this cubic, gives a positive root $\chi=2.1099$, and $\lambda=64^{\circ} 38' 30''$.

$\therefore h=156^{\circ} 16' 4''=10$ hours, 25 minutes, 4 seconds, and $2h=20$ hours, 50 minutes, 8 seconds, length of day.

II. Solution by JOHN M. ARNOLD, Crompton, R. I.

In the triangle ABC formed by the zenith, the pole, and the sun when crossing the prime vertical, $A=90^{\circ}$, B =the hour angle, a =sun's polar distance, and c =the co-latitude.

In the triangle $A'B'C'$ formed by the north point of the horizon, the pole, and the sun when rising, $A'=90^{\circ}$, B' =supplement of the hour angle at rising, c' =the latitude, and $a'=a=90^{\circ}-\delta=66^{\circ} 32' 45''$.

From trigonometry, $\cos B=\cot a \tan c \dots\dots(1)$; $\cos B'=\cot a' \tan c' \dots\dots(2)$; $c'+c=90^{\circ} \dots\dots(3)$. From the conditions of the problem, $B'=180^{\circ}-2B \dots\dots(4)$. From (2) and (3), $\cos B'=\cot a \cot c$. Eliminating $\cot c$ by (1), $\cos B'=\cot^2 a / \cos B$. Eliminating $\cos B'$ by (4), $2\cos^2 B-1=-(\cot^2 a / \cos B)$. Putting $\cos B=y$ and substituting the numerical value for $\cot^2 a$, $y^3=\frac{1}{2}y+.094118=0$. Finding the two positive roots of this equation, $y=.5815$ and $.2056$. From the first root $B=54^{\circ} 27'=3$ hours, 37 minutes, 48 seconds. Length of day= $4B=14$ hours,

31 minutes, 12 seconds. From the second root, $B=78^{\circ} 8' = 5$ hours, 12 minutes, 32 seconds. Length of day = 20 hours, 50 minutes, 8 seconds.

Values of B substituted in (1) give for the latitude $36^{\circ} 44'$ and $64^{\circ} 39'$.

III. Solution by C. HORNING, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O.

Let Z be the zenith, P the pole, N the north point of the horizon, S the place on the horizon where the sun rises, and S' the place of the sun on prime vertical when it has moved half the way to the meridian. Also let ϕ = the required latitude, δ = the sun's declination = $23^{\circ} 27' 15''$, and P = the hour angle of the sun when rising. Then without allowing for refraction and semi-diameter, we get from the spherical triangles PNS and PZS' , $-\cos P = \tan \phi \tan \delta \dots (1)$; and $\cos \frac{1}{2}P = \cot \phi \tan \delta \dots (2)$. From (1) and (2) we have $\tan^2 \phi - (1/\tan \delta) \tan^2 \phi + 2 \tan \delta = 0$, or $\tan^2 \phi - 2.31224 \tan^2 \phi - .18704 = 0$. Whence $\tan \phi = 2.27614$, and $\phi = 66^{\circ} 16' 54''$ the required latitude. From (2) $P = 158^{\circ} 1' 28''$. Hence from sunrise to noon is 10 hours, 32 minutes, 5.8 seconds, and the length of the day is 21 hours, 4 minutes; 11 seconds.

Solved by J. SCHEFFER with result, latitude $36^{\circ} 43' 31''$, and length of day 14 hours, 31 minutes, 6 seconds.

61. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

The product of n numbers, each the sum of four squares, may be expressed as the sum of four squares in $(48)^{n-1}$ different ways.

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

In Vol. II, No. 2, page 47, it is demonstrated that the product of two numbers each the sum of four squares may be expressed as the sum of four squares in 48 different ways.

Let $a_1^2, a_2^2, a_3^2, a_4^2 \dots a_n^2$ be the n squares.

Then $a_1^2 a_2^2 = (m_1^2 + m_2^2 + m_3^2 + m_4^2)$ in 48 ways.

$$a_1^2 a_2^2 a_3^2 = (m_1^2 a_3^2 + m_2^2 a_3^2 + m_3^2 a_3^2 + m_4^2 a_3^2) = (n_1^2 + n_2^2 + n_3^2 + n_4^2) \\ \text{in } 4 \times 48 \times 48 = 2^2 \cdot 48^2 \text{ ways.}$$

$$a_1^2 a_2^2 a_3^2 a_4^2 = (n_1^2 a_4^2 + n_2^2 a_4^2 + n_3^2 a_4^2 + n_4^2 a_4^2) = (o_1^2 + o_2^2 + o_3^2 + o_4^2) \\ \text{in } 4 \times 2^2 \cdot 48^2 \times 48 = 2^4 \cdot 48^3 \text{ ways.}$$

$$a_1^2 a_2^2 a_3^2 a_4^2 a_5^2 = (p_1^2 p_2^2 p_3^2 p_4^2) \text{ in } 4 \times 2^4 \cdot 48^3 \times 48 = 2^6 \cdot 48^4 \text{ ways.}$$

$$\therefore a_1^2 a_2^2 a_3^2 a_4^2 \dots a_n^2 = (z_1^2 + z_2^2 + z_3^2 + z_4^2) \\ \text{in } 2^{2n-4} \cdot 48^{n-1} \text{ ways} = (2)^{2(n-2)} \cdot (48)^{n-1} \text{ ways.}$$